THREE DIMENSIONAL MAGNETOHYDRODYNAMICS OF FUSION PLASMAS:

STUDY OF MAGNETIC TOPOLOGY IN HELICAL RFP STATES

Marco Veranda

October 5, 2010

European Joint PhD in Fusion Science and Engineering
Reversed Field Pinch configuration and experimental results
• **RFP is an axisymmetric toroidal configuration** for the magnetic confinement of fusion plasmas;

• **toroidal magnetic field:**
  • mainly generated by currents flowing in the plasma;
  • reverses its sign in the outer region.
• At $I_P < 1$ MA the plasma stays in Multiple Helicity (MH) states;

• the presence of several MHD instabilities (resistive kink-tearing modes) and the overlapping of magnetic island results in a chaotic magnetic field.
• At $I_p < 1$ MA the plasma stays in Multiple Helicity (MH) states;

• the presence of several MHD instabilities (resistive kink-tearing modes) and the overlapping of magnetic island results in a chaotic magnetic field.
• At higher current ($I_p > 1$ MA) the amplitude of the innermost resonant mode increases while the secondary modes decrease;

• **magnetic transition to Quasi Single Helicity states (QSH);**
At higher current ($I_p > 1$ MA) the amplitude of the innermost resonant mode increases while the secondary modes decrease;

- magnetic transition to Quasi Single Helicity states (QSH);
A topological transition in the magnetic field configuration is observed:

- separatrix expulsion process, the island X-point merges with the axis-symmetric O-point;

![Double Axis (DAx)](image1)

![Single Helical Axis (SHAx)](image2)
A topological transition in the magnetic field configuration is observed:

- separatrix expulsion process, the island X-point merges with the axis-symmetric O-point;

**FORMATION OF INTERNAL TRANSPORT BARRIER (ITB)**
• A **topological transition** in the magnetic field configuration is observed:

  - **separatrix expulsion** process, the island X-point merges with the axis-symmetric O-point;

  ![Diagrams of Double Axis (DAx) and Single Helical Axis (SHAx)]

• **A helical equilibrium situation** is experimentally diagnosed, with a **strong reduction of magnetic chaos**
• **PART I:**
  
  - study of the magnetic topology of quasi-helical RFP states resulting from 3D magnetohydrodynamic (MHD) simulations;

• **PART II:**
  
  - study of the magnetic topology in the ITB surrounding the helical core of RFX-mod plasma, with data from gyrokinetics simulations; $\chi_e$ estimate.
Topological study of RFP states coming from 3D MHD simulations
• The study of RFP configuration is pursued through the **visco-resistive MHD** model;

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} - \eta \mathbf{J} \right)
\]

• Nonlinear numerical simulation by 3D cylindrical and spectral SpeCyl code:

\[
\frac{\partial \mathbf{v}}{\partial t} = \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v}
\]

\[
\nabla \times \mathbf{B} = \mathbf{J}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

- constant and uniform density \( \rho \);

- zero pressure (neglecting plasma \( \beta \)).
• From the magnetic point of view, transition from MH to Single Helicity (SH) states was found as system dissipation ($\eta,\nu$) is increased;

\[ H = (\eta \nu)^{-\frac{1}{2}} \]
• **NEMATO** integrates the magnetic field lines differential equation \( \frac{dx}{d\tau} = B(x) \):
  - given the B field on a 3D grid;
  - in cylindrical or toroidal geometry;
  - using a **volume preserving algorithm**, that respects the \( \nabla \cdot B = 0 \) constraint;

* Finn, Chacon, Phys. Plasmas **12**, 2005
• The geometry used in the following simulations is a cylindrical one:
  - periodical boundary conditions on the $\theta,z$ coordinates;
  - aspect ratio $R/a=4$.

• NEMATO gives the intersection of the field lines with Poincaré surfaces of section.
**MAGNETIC ORDER**: field lines lie on a surface (magnetic surface) whose normal is always perpendicular to magnetic field;

**MAGNETIC CHAOS**: a magnetic field line fills a finite plasma volume.
• NEMATOTO has been validated against a SH RFP equilibrium:
  
  – helical symmetry: all quantities depend on two variables, $r$ and $u=m\theta-nz/R_0$;
  
  – magnetic field lines lie on magnetic surfaces;
  
  – magnetic surfaces can be analytically described by the helical flux function $\chi(r,u)$:

\[
\chi(r,u) = mA_z + nrA_\theta / R_0 \quad , \quad B(r,u) \cdot \nabla \chi(r,u) = 0
\]

\[
\chi(r,u) = \text{cost} \quad \text{defines a flux surface}
\]
• **SpeCyl simulation of a stationary SH equilibrium** with:
  \[ S = \frac{1}{\eta} = 3 \cdot 10^4, \quad P = \frac{\nu}{\eta} = 300, \quad \Theta = 1.5 \]

• **χ levels** by SpeCyl and Poincaré maps by NEMATOTO agree (using adequate grid resolution).

(a) low resolution: \( n_r \times n_\theta \times n_z = 100 \times 256 \times 256 \)

(b) standard resolution: \( n_r \times n_\theta \times n_z = 100 \times 256 \times 2048 \)

(c) differences between the two cases.
NEMATO has been applied to the study of a full 3D simulation (no symmetry):

\[ S = \frac{1}{\eta} = 3 \cdot 10^4, \quad P = \frac{\nu}{\eta} = 1000, \quad \Theta = 1.5. \]

- initial perturbation: resonant \( m=1 \) modes, \( n=-9 \) / \( n=-10 \);
- nonlinear evolution: QSH state with \( m=1 \), \( n=-9 \).
• $t=500 \ \tau_a$
• before island overlap;
• mode $n=-9$ and $n=-10$ separatrix calculated from helical flux functions;
• effect of mode interaction;
ISLAND OVERLAP/2

- $t=600 \tau_a$
- after island overlap;
- chaos between the two island chains.
- \( t = 710 \, \tau_a \)
- before separatrix expulsion;
- increased chaotic region;
• $t=750 \tau_a$
• *after* separatrix expulsion;
• *cylindrical* axis disappeared;
• increase of the region with conserved surfaces;
- $t = 790 \tau_a$
- after separatrix expulsion;
- chaos healing almost complete;
QSH STATE

- $t=1800 \tau_a$
- $n=-9$ mode nonlinearly saturated;
- QSH / SHAx state;
- chaos healing even with larger secondary modes amplitude;
Topological study in ITB’s region with data coming from gyrokinetic simulations
INTERNAL TRANSPORT BARRIERS (ITB)

- SHAx states:
  - onset of an ITB;
  - presence of strong gradients of electron temperature
MICROTEARING MODES

• Strong gradients are reservoir of free energy available to trigger microinstabilities;

• gyrokinetik simulations (GS2) show that Micro-Tearing (MT) modes are unstable at the barrier (due to temperature gradients);

• the action of saturated MT perturbations is predicted to lead to magnetic field lines stochastization and to enhanced energy diffusion.
AIM OF THE WORK

- Study of the magnetic topology in the ITB's region;
- estimate of electron thermal conductivity $\chi_e$.

1) TOPOLOGICAL STUDY OF THE MAGNETIC FIELD IN THE ITB

2) DEFINITION OF THE TRANSPORT COEFFICIENTS

3) CALCULATION OF THE ELECTRON THERMAL CONDUCTIVITY
\[ B(r) = B_0(r) + b(r) \]

- The macroscopic equilibrium field \( B_0(r) \):
  - cylindrical symmetry;
  - obeys magnetostatic MHD equation:
    \[ \nabla \times B_0(r) = \sigma B_0(r) \]
• The **perturbed** field is Fourier decomposed in $\theta,z$ directions:

$$b = \sum_{mn} b_{mn}(r) \cdot \exp[i(m\theta - nz)]$$

• perturbed modes characteristic, depending on $(m,n)$, are obtained from the gyrokinetic code.
• Poincaré surface of section;

\[ z = 2\pi R_0 \]

\[ z = 0 \]
The description of electron thermal conductivity in a stochastic magnetic field relies on the definition of the magnetic diffusion coefficient $D_m$:

$$D_m = \frac{1}{2} \frac{d}{dL} \langle (\Delta r(L))^2 \rangle$$

$$\chi_e = \frac{1}{2} \frac{d}{dt} \langle (\Delta r(L))^2 \rangle = D_m \cdot v_{th,e}$$

- $r$: direction perpendicular to the equilibrium magnetic field flux surfaces;
- $L$: magnetic field lines’ arclength;
- $v_{th,e}$: electrons’ thermal velocity.
• $D_m$ can be deduced considering a random-walk diffusion process of test particles following the stochastic magnetic field lines:

$$D_m^{RW} = \frac{\langle (\Delta r)^2 \rangle}{2L}$$

CALCULATED BY NEMATO

• An upper limit for electron thermal conductivity can be given considering the collisionless estimate*:

$$\chi_e = D_m^{RW} \cdot V_{th,e}$$

RESULTS

• Poincaré surface of section at $\theta=0$;

• $D_m$ is:

$$D_m = 1.4 \cdot 10^{-6} \text{ m}$$

• Considering 1 keV electrons:

$$\chi_e \approx 20 \text{ m}^2\text{s}^{-1}$$

• Well compared with experimental data.
• Study of magnetic topology in helical RFP states:

  – global tearing modes:
    • first application of the code to the study of the chaos healing effect confirms the role of separatrix expulsion process (more sistematic study is ongoing);

  – microtearing modes:
    • considering a stochastic magnetic field due to MT perturbations a value of electron thermal conductivity in agreement with experimental data was found.
FUTURE WORK

• MHD studies of RFP self organization considering:
  – toroidal effects;
  – self-consistent temperature evolution;
    possible through
  – extended MHD code PIXIE3D*;
