IDENTIFICATION OF LOCAL ALFVÉN WAVE RESONANCES WITH REFLECTOMETRY AS A DIAGNOSTIC TOOL IN TOKAMAKS

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Local Alfvén wave (LAW) resonances are excited in tokamaks by an externally driven electromagnetic field, below ion cyclotron frequency. We show that combination of small power deposition in LAW resonances, swept by plasma density variation or scanned by varying generator frequencies, in combination with detection of the density fluctuations in the LAW resonances by reflectometry can serve as diagnostic tool for identification of the effective ion mass number $A_i$, and $q$-profile in tokamaks. The idea is based on the simultaneous detection of the position of $m=\pm 1$ local AW resonances, which are excited by $M/N=\pm 1/\pm 2$ antenna modes, and $m=0$ generated by poloidal mode coupling effect in tokamaks. The $m=0$ resonance depends only on the effective ion mass number and does not depend on the $q$-profile, so that the mass number can be determined unambiguously. Then, we can determine $q$-factor at the position of $m=\pm 1$ LAW resonances. Using multifluid ALTOK code, we identify mass number in TCABR experiments and demonstrate the possibility of applying this method in Joint European Torus.

The idea of strong resonant absorption of RF fields, which are excited at the local Alfvén wave (LAW) resonance in magnetically confined inhomogeneous plasmas, is based on a series of the theoretical works carried out in seventh and eightieth (for example, [1-3]). LAW resonance excitation is defined by mode conversion of an externally driven RF field below the ion-cyclotron frequency, $\omega_{ci}$, into the kinetic or electrostatic Alfvén wave (shear AW) at the Alfvén resonance layer where the resonance density can be determined by the equation in “cylindrical” form

$$k_z^2 = \frac{\omega^2}{c_{Ai}^2} \left[ \frac{A_i n_i}{n_e} + \frac{A_z n_z}{n_e} \right] \frac{1}{1 - \frac{\omega^2}{\omega_{ci}^2}} + \frac{\omega^2}{\omega_{ci}^2}$$

$$k_z = \frac{B_t}{B_0 R_0} \left( N + \frac{m}{q(r)} \right)$$

$$c_{Ai} = \frac{B_0}{\sqrt{\mu_0 m_i n_e}}$$

where $m$ and $N$ are toroidal and poloidal wave numbers, $q$ is the safety factor $B_0$, $B_t$ are the modulus and toroidal magnetic field, and $A_z$ is the main impurity mass number. Eq. (1) can be reduced to $\omega_{ci} = c_{dd} k_z << \omega_{ci}$ in the low frequency limit, where $c_{d} = c_{dd}/\sqrt{2 A_i}$ is the Alfvén velocity, $A_{d/ef} = (A_i n_i + A_z n_z)/n_e$ is effective mass number, and $\omega_{ci}$ is the ion cyclotron frequency. The continuous spectrum (1) of the LAW resonances is known as the Alfvén wave continuum. In the standard quasi cylindrical model for wave excitation in tokamak plasmas, the oscillating RF field is represented as a sum of harmonics $\exp[i(m0+N\Phi - \omega t)]$. These fields are excited in the plasma by an antenna current sheet, $J_{0,\phi}$ ($r,\theta,\phi, t$) = $\Sigma_M J_{0,\phi} (M,N) \delta(r-b) \exp[i(M0+N\Phi - \omega t)]$. The antenna spectra of $J_{0,\phi} (M,N)$ driven in tokamaks are usually very wide both for the toroidal and poloidal wave numbers; however, a few harmonics can satisfy the LAW resonance conditions (1). Due to toroidal effects, the poloidal wave numbers $m$ in the plasma may be different from $M$ that are excited by the antenna. In particular, although $m=0$ LAW resonance can not be directly driven by an external antenna, poloidal mode coupling allows it to be excited as a satellite mode of $|m|=1$ LAW resonance, in tokamak plasmas. Indeed, that has been already demonstrated experimentally in TCA with a CO$_2$ laser interferometry [4] when it was operating in Lausanne. Recently, power deposition in $m=0$ local AW resonance has been also demonstrated by ECE heterodyne system in TCABR [5].

Moreover, AW can also be excited in a plasma as global Alfvén waves (GAW) [1-2] and toroidicity induced Alfvén eigenmodes (TAE) at the position of bifurcation of the LAW.
resonances, for given \( N \) and different \( m \neq 0 \) numbers, with wave fields corresponding to the discrete eigenfunctions of the proper boundary value problem. Easy identification of these modes were explored as a diagnostic tool for definition of the effective ion mass number \( A_{\text{ef}} \) and \( q \)-profiles in a series of tokamak experiments (for example, [5-11]). This technique, called Magneto-Hydro-Dynamic (MHD) spectroscopy, is usually based on external measurements by magnetic probes and work in the active (AW excitation by an external antenna) and passive (using AW instabilities driven by accelerated ions in NB or ICR heating) regimes.

Fig. 1. Time traces (70-97 ms) of the loop voltage and line averaged density (a), toroidal current (b), and amplitude and phase density oscillation measured with reflectometer (c) in the TCABR discharge with AW heating. The loop voltage in a control ohmic discharge \# 10669 is also shown in (a), for comparison.

Recently, using a fixed frequency (32.4GHz) O-mode reflectometer, wave driven density fluctuations at the local LAW resonance \( m=\pm 1, N=\pm 2,3 \) with frequency \( f_{\lambda} = 4 \text{ MHz} \) and relatively small power deposition of 30 kW were detected in the Tokamak Chauffage Alfvén Brésilien (TCABR) [12]. The LAW resonances were excited by a fixed AW frequency generator, which usually used for heating, together with a small density increase in these experiments. However, this density increase during RF pulse does not produce a relative sweeping of reflectometer reflection point over LAW resonance position, as follows from eq. (1)

\[
n_A = \left[ \frac{B_j}{4\pi m_i\omega R_0} \left( \frac{\omega}{\omega_{\text{ci}}} \right)^2 \left( \frac{n_i}{n_e} + \frac{A_{\lambda} n_z}{n_e} \right) \right]^{-1} \frac{n_{\text{ref}}}{1 - \omega^2 / \omega_{\text{ci}}^2} ; n_{\text{ref}} = \pi \frac{m_e}{e^2} f_{\text{ref}}^2
\]

Here \( n_{\text{ref}} \approx 1.3 \cdot 10^{13} \text{ cm}^{-3} \) is the density at the reflection point, and \( n_A \) is the one at LAW resonance position. In Eq.(2), \( q \) factor and an impurity density variation may only allows to meet the condition \( n_A = n_{\text{ref}} \).

In this paper, based upon results of calculations carried out with the two dimensional multi-fluid ALTOK code [13] and taking into account that the \( q \) variation was rather small during the RF pulse, we show that the average mass number could be determined in the experiments [14], and we discuss the possibility of frequency sweep of \( m=0, \pm 1, N=\pm 2 \) LAW resonances, within the frequency band 0.5-0.9 MHz, to explore this effect in JET.
Fig. 2. Plot of Alfvén radial density profile $n_A$ for $A_{ef}=1.1$, $q_0=1.07$, $m/N=-1/3$ mode (dash-dot-dot line), and for $A_{ef}=1.17$, $q=1.08$, $m/N=-1/-2$ (solid line) electron density distributions for $n = 1.2 \times 10^{19} \text{ m}^{-3}$ (dashed) and $n = 1.5 \times 10^{19} \text{ m}^{-3}$ (short dashed line), the reflectometry cut off density in TCABR is marked by dotted line (a); amplitudes of the AW density fluctuations calculated with ALTOK for 4 MHz frequency $m/N=-1/3$ mode (dash-dot-dot line, respective to (a), and $m/N=-1/-2$ (solid line) resonance for $n = 1.5 \times 10^{19} \text{ m}^{-3}$ (b).

Plasma Model of TCABR and JET
The calculations with the ALTOK code have been carried out assuming circular, in TCABR, and D-shape cross-section ($\kappa=1.43$, $\delta=0.25$) in JET. To obtain good accuracy, 473x99 mesh points are used in the numerical calculations. Here, we analyze the AW absorption in TCABR shot N°10668 [11] (minor radius $a=0.18\text{m}$, major radius $R_0=0.615\text{m}$, toroidal magnetic field $B=1.15\text{T}$, plasma current $I_p=78-80\text{ kA}$, with safety factor in the center $q_0=1.1$, line averaged plasma density $\bar{n}=1.2-1.5 \times 10^{19} \text{ m}^{-3}$ (gas hold hydrogen), central electron and ion temperatures $T_e=450 \text{ eV}$ and $T_i=150\text{eV}$). The TCABR antenna module has two groups of RF current carrying straps, which are positioned in two toroidal cross-sections separated toroidally by an angle about $22^\circ$, creating mainly the spectrum of the poloidal $M=\pm 1, \pm 2$ and toroidal $N=\pm 1, \pm 2, \pm 3$... modes. Their LAW resonances are strongly separated by choosing the generator frequency 4 MHz. In JET calculations, we use shot #60895 as a reference (minor radius $a=1.05\text{m}$, major radius $R_0=2.85 \text{ m}$, toroidal magnetic field $B=1.0 T$, plasma current $I_p=1.2 \text{ MA}$, safety factor $q(0)=1.1$, central plasma density $n_0=5\times 10^{19} \text{ m}^{-3}$ (gas hold deuterium), central electron and ion temperatures $T_e=3000 \text{ eV}$ and $T_i=2000 \text{ eV}$, respectively. The generator frequency has been swept in the band $f=0.2-0.5 \text{ MHz}$. The antenna module is proposed to have two groups of RF current carrying straps. These groups are situated at two opposite toroidal positions creating mainly the spectrum of the poloidal $M=0, \pm 1, \pm 2$ and toroidal $N=\pm 2, \pm 4$ modes. The plasma profiles used in the code calculations are quasi parabolic temperature profile $T=T_0 \left(1-\Psi\right)^{0.9}$, density profile of TCABR $n=n_0(1-\Psi)^{0.7}$, and of JET $n=n_0(1-\Psi)^{0.4}$, and current profile is $j=j_0(1-\Psi^{0.9})^{1/2}$, where $\Psi$ is the normalized poloidal magnetic flux.
Identification of Local AW Resonance with Reflectometry

The density fluctuation rise in the LAW position can be detected by a reflectometer (for example, Ref.16-17). In the simple case, the relative amplitude of the density fluctuations $\delta n/n$ is proportional to the parallel component of the electric field, in accordance with Boltzmann distribution, and to the square root of the absorbed power density $\tilde{p}$,

$$\frac{\delta n}{n_e} = k_e e^2 \frac{\tilde{E}_\parallel}{4\pi v_e} \approx \frac{e\tilde{E}_\parallel}{m_e k_e v_e^2} \left| \tilde{E}_\parallel \right| = 2 \sqrt{\frac{8\pi}{\pi \omega_e^2}} \frac{k_e v_e^2}{\omega_e} \tilde{p} \tag{3}$$

where $n_e$, $v_e$, and $\omega_e$ are the electron mass, thermal speed, and plasma frequency, respectively. For power density of 0.2kW/m$^3$ at the resonance surface $r_s=0.65(R-R_0)$ (or about of 1kW of total absorbed power), we obtain $\delta n/n_e=1.10^{-4}$, which is of the level that can be detected by a modern reflectometer (for example, Ref.13). The sensitivity of the system may be improved with locked frequency detection related to the AW frequency. In Fig.1, we show the time traces (70-100 ms) of the loop voltage, line averaged density, toroidal current, and respective amplitude of 4MHz oscillating phase measured with the 32.4 GHz O-mode reflectometer in TCABR discharge # 10668 with AW heating. The density trace during RF pulse was in accordance with the variation of the bolometry signal $\propto Z_{\text{eff}}^4 n^2/T^{3/2}$ assuming that there is small $Z_{\text{eff}}$ rise from 1.5 to 1.8, as found from ASTRA calculations, using the small loop voltage increase $\propto Z_{\text{eff}}/T^{3/2}$ and $\beta_{\text{eq}}$ value at the beginning of the RF pulse (diminishing later) in comparison with the similar ohmic discharge # 10669. We observe that, consistently with the calculated increase in $Z_{\text{eff}}$, the loop voltage for discharge # 10668 shows a strong increase well correlated with the first spike in the density oscillation detected by reflectometer (the negative loop voltage spike at $t = 86$ ms is due to converting of different load resistors in the ohmic circuit to extend the plasma current). The central electron temperature does not increase during the RF pulse, as indicated by the ECE signal in this experiment. In Fig.1c, the observed maxima of the amplitude of 4 MHz microwave phase should be interpreted as local AW resonances for $m/N=-1/3 (A_{\text{ef}}=1.1)$, in the initial stage, and as a superposition with $m/N=-1/-2 (A_{\text{ef}}=1.17)$ in the final stage of the RF discharge # 10668.

Alfvén Wave Absorption in JET

The AW continuum is calculated for JET, using ALTOK code and parameters already specified. The results for $m=0$, $\pm 1$, $N=-2$ are shown in Fig.3(a) It is specially relevant to note that the continuum line for $m=0$ extends most of the inner region of plasma, for the 0.5- 1.0 MHz frequency band, offering the possibility to determine the effective mass number in JET through the detection of the LAW resonances by a reflectometry. Another possibility is related to detection of the $|m|=1$ LAW resonance. In Fig.3 b and c, we show specific dissipation profiles for these cases, as an example. The respective absorption profiles for 800 kHz and for 720 kHz frequencies are presented in Fig. 3 b and c, respectively, being both excited by the $M=-1$, $N=-2$ antenna. The absorption in Fig.3c is due to the GAW resonance of the $m/N=-1/-2$ mode. There is a correlation of the local AW resonance position of the $N=-2$ and $m=0$ in Fig.3a with the first absorption spike at $R-R_0=0.8 \ a$ in Fig.3c, calculated for the $m=0$ mode. A typical absorption in the $m/N=-1/-2$ AW continuum is shown in Fig.3b for 800 kHz frequency. The power deposition profiles corresponding to these cases are shown in Fig.4. The half width of those spikes are 3% of the plasma minor radius, i.e., smaller than 4cm, and therefore offering good possibility for identification of the resonance position by reflectometry.

Discussion

To find out the effective mass number, the variation of the $m=0$ local AW resonance position, in accordance with sweeping the AW frequency, should be mapped by scanning
the reflectometry frequency, to identify the local AW density $n_\text{LAW}(r)$. Then, using the cylindrical equation (2) simplified for $\omega \ll \omega_{ci}$, we have

$$A_{ef}(r_j) = \frac{1}{4\pi m n_\text{LAW}(r_j)} \left[ \frac{B_0}{2\pi f_j R_0} \left( \frac{N + \frac{m}{q(r_j)}}{} \right) \right]^2$$

(5)

where $B_0$ is the toroidal magnetic field on the magnetic axis, $m_i$ is mass of the main plasma species, $f_\text{i}$ is the generator frequency at the moment of identification of the LAW resonance position, and $n_\text{LAW}$ should be averaged over the magnetic surface where the LAW resonance is detected. A more precise option is to calculate AW continuum curve using exact equilibrium parameters and the main plasma species mass with the ALTOK code (like shown in Fig.3a) and compare them with the code calculations using the $n_\text{LAW}$ distribution over radius as found from the reflectometry measurements. Then, using the ratio of one curve to the other, we obtain the value of $\sqrt{A_{ef}(r)}$. Further, identifying radial position of the $m=-1$, $n=-2$, 3 AW resonances, we can find the cylindrical $q$ from Eq. (5), which should be adjusted with the code calculations for different neighbor $q$ profiles in the next step. Here, we note that identification of $m=0$ LAW spectrum has strong advantages for calculation the effective mass number in comparison with the other modes because the $m=0$ AW continuum does not depend on $q$-factor and it is not interrupted by TAE modes for low $m$.

**Conclusion**

- Using ALTOK code we find the mass number $A_{ef}=1.1-1.17$, in fixed frequency AW heating experiments in TCABR, where the density at the $m=-1$, $n=-2$, 3 local AW resonances was identified with a fixed frequency O-mode reflectometer.
- Calculations with ALTOK code show that AW frequency sweeping, in the band 0.2-0.5 MHz for $m=0,-1$, $N=-2$, accompanied by scanning the frequency of reflectometer, is quite viable scheme to identify the LAW resonance positions over the major part of the plasma cross section. Localization of the $m=0$ AW continuum helps to identify the effective mass number and $q$ profile can be found from $m=\pm 1$ AW continuum in JET.
- Finally, we conclude that a combination of sweeping the frequency of the AW excitation system with sweeping of the reflectometer frequency may be a very powerful diagnostic tool to find effective mass number $A_{ef}$ and $q$-profiles in tokamaks.

**References**

Fig. 3. Plot of absorption profile for TAE mode in the gap $f=256.9-262.9$ kHz of $m/N=-1/2,-4/2$ continuum (a), distribution of AW continuum frequency together with the position of some separate spikes of absorption for $m=0,-1, N=2$ (b), and absorption profile in the continuum band $f=380-406.5$ kHz (c) in JET.