On the difference between phase and amplitude behaviours: dependencies on the radiation pattern and on the turbulence level

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Abstract: The behaviour of the phase and amplitude needs to be explored to improve the interpretation of the measurements done especially on the different behaviours of the phase and amplitude extracted from the reflectometry measurements. This problem is connected to the radiation pattern properties, which permits to integrate the reflected signal on a given range of probing angles. The previous studies have done considering wave in vacuum reflected on corrugated surface [1-2]. This method has been also used to study the asymmetry in the received signal [3]. Some 2D full-wave simulations have permitted to show that destructive interference can take place and drive signal extinctions [4]. Recently some data processing has been done on amplitude and phase measurements and compared [5]. The cross-correlation lengths of the phase and amplitude are different and seem in experiments to be lower than expected. A possible explanation will be presented taking into account the role of the radiation pattern and based on the ray tracing that takes into account the wave propagation into the plasma in a simpler manner than 2D Full-wave can do. At first, the description of the reflectometer using multi-beam ray tracing is presented, and then the choice done to recombine the rays to mimic the receiver, which permits to study the recombination of the rays as function of the radiation pattern properties. This part revisits an old question on the use of the narrow or wide beam, which permits in the framework of this work to say that a better correlation exists between phase and amplitude in the case of narrow beam while the phase difference of the reflected waves stays much smaller than \( \pi \) weighted by the distribution of the phase captured rays. This phenomenon follows simply the Physics of extended source with a phase modulation introduced by density perturbations or turbulence even if the density fluctuations are not at the cut-off (forward scattering effects). Here just the impact of the index effects are taken into account, the introduction of the Bragg scattering requires a full-wave computation. A discussion on the possible uses of such effects is presented for extracting turbulence properties especially within the recent developments of synthetic antennas where the radiation pattern can be versatile to be adapted to magnetic topology or as it is on MAST to realize 2D Doppler reflectometry measurements [6].

Introduction: The aim of this work is done to complete the knowledge of the role of the radiation pattern in the reflectometry measurements where it is often admitted that the phase and the amplitude correspond directly to those of the reflected waves excepted in few cases as mentioned in [4]. Usually in simulation these effects are taken into account by the introduction of a horn as a receiver [4] or using an integral to evaluate the phase and the amplitude of the received wave with a Gaussian beam weighting to describe the properties of the receiver, but without a precise idea on the connection between the phase and amplitude on the reflected waves and the measurements of it quantities. The time-dependent effects of the fast variations of the density fluctuations are here not considered but results can in found in [7-8] which need to solve wave equation to access to these time-dependent effects as frequency shifts and amplitude variations. The first tool use to evaluate the links between the density perturbation and phase-amplitude of the detected signals was based on the optical behaviour of an electromagnetic wave reaching a corrugated mirror usually used to interpret the radar signal reflected on random rough surfaces [9]. The presented theories in this book are also able to describe the depolarization of the probing wave, which strictly speaking has to be considered to study the amplitude variations of a detected reflectometer signal. The full-wave simulation is the more relevant tool to do this but the computation time limits the number of the considered cases. However the ray tracing permits to response to the main questions on this subject. Knowing that the scattering effects are ignored here but the scattered waves drive the same effects as those studied here only their interpretations remained different.

We describe first the model associated to the reflectometer simulation and what are the rules used to implement the phase and the amplitude measured by a receiver. Then test cases are designed to validate
the choice done to describe the behaviour of a reflectometer as function of its radiation pattern with and without plasma. During these studies we have shown that the Physical Optical Model is not well adapted to determine the dependencies of reflectometry signals working in plasmas. An important parameter has been highlighted: the spatial evolution of the phase along the antenna mouth of the detected wave. This can be reduced in the following criterion: If the phase variations, between sub-beams describing the reflected wave, along the antenna mouth is greater than $2\pi$ for a large number of sub-beams, the interpretation of the received signal becomes complex or even impossible. A set of representative events, happening in tokamaks, is examined showing the link between the measured wave amplitude and the measured phase. These studies drive a recommendation: a narrow radiation pattern is the best for reaching relevant measurements of the phase and amplitude of the received wave provided by a reflectometer, but this increases the probability to loose the reflectometer signal.

1) Context and Tool Description:

A one-dimensional description imposes that the electromagnetic flux should be conserved, thus the amplitude variations can only come from the superimposition of reflected and scattered waves, and the measured phase is a combination of the phases of different waves. As consequence the amplitude and phase are linked that drives interferences. However at low level of density perturbations this link has been described in the framework of Born approximation [10]. This link for higher level of density perturbation is more difficult to establish due to wave trapping [11-13] and multi-scattering effects [14-15]. The aim of this paper is to improve the knowledge of the 2D effects on the phase and amplitude measurements, and to determine what are the most appropriate numerical tools able to describe accurately enough these 2D effects. The radiation patterns of the emitter and of the receiver have to be taken into account, that requires to choose how to describe it to reach easily the studied parameters phase and amplitude of the backward waves. Different tools exist the simplest is the Physical Optics Modelling, which considers just a corrugated or random rough surfaces and a wave propagation in vacuum [1,9]. A more sophisticated model corresponds to the ray tracing, which integrates the wave propagation in plasmas under the WKB approximation [16] and the introduction of sub-beams permits to evaluate the spatial phase distribution at the antenna mouth, which appears to be an important parameter from this work. The use of full-wave codes is the most relevant but its computation time requirement restricts its use. This is one the main reason why these 2D effects on the phase and amplitude measurements have been done using a ray tracing code. The second one is the extraction of the spatial phase variations at the antenna mouth when a metallic horn is introduced in the simulation, which is not the case when Huygens surfaces are used in the case of free propagation. So, only the refractive index effects are taken into account in such description, except if an appropriate scattering model adding sub-beams or mode conversion is associated to the ray tracing code [17].

a) Description of the numerical tools:
The ray tracing code is based on the standard modelling, which consists to solve a set of coupled Ordinary Differential Equations (ODE) written as following:

\[
\begin{align*}
\frac{\partial \mathbf{r}}{\partial \tau} &= \frac{\partial D(\omega, \mathbf{k}, \mathbf{r}, t)}{\partial k}, \\
\frac{\partial \mathbf{k}}{\partial \tau} &= \frac{\partial D(\omega, \mathbf{k}, \mathbf{r}, t)}{\partial \mathbf{r}}, \quad \text{and optional for time-dependent cases} \\
\frac{\partial \omega}{\partial \tau} &= \frac{\partial D(\omega, \mathbf{k}, \mathbf{r}, t)}{\partial \omega}, \\
\frac{\partial t}{\partial \tau} &= -\frac{\partial D(\omega, \mathbf{k}, \mathbf{r}, t)}{\partial t},
\end{align*}
\]

where D correspond to the dispersion equation for the considered mode, here O (ordinary) or X (extraordinary)-mode. This set of ODE is solved using an adaptive time-step Runge-Kutta scheme including a constraint forcing to fulfil the dispersion relation. As the extracted parameters are the phase and the amplitude, we choose to define to distribute the radiated energy on equal energy rays with an angular distribution following the considered radiation pattern putting the same among of energy on each ray. The phase is obtained by computing the WKB phase along the ray trajectory defined as following:

\[\Phi_j = \int_{x_{in}}^{x_{out}} \mathbf{k}_j \cdot d\mathbf{s}\]

where \(x_{in}\) and \(x_{out}\) are the position vectors at the exit and entry of one ray constituting the probing beam.
The measured phase is the result of average phase $\phi_j$ of the rays reaching the antenna mouth. The simple definition comes from the fact each rays has the same energy contain that is to say the same weight. That can be corrected by introducing a weighting function $w_j(\theta_j)$ for the receiving radiation pattern, which can be different than the emitter one. The incident angle $\theta_j$ is determined using the normal vector to antenna mouth and the wavenumber vector reaching the surface corresponding to the antenna mouth. The measured phase can be defined by:

$$\phi_m = \sum_{j=1}^{n_{max}} w_j(\theta_j) \phi_j.$$

The measured wave intensity follows the definition

$$I = \sum_{j=n_{min}}^{j=n_{max}} a_j e^{i \phi_j} * \sum_{j=n_{min}}^{j=n_{max}} a_j^* e^{-i \phi_j},$$

where $n_{min}$ and $n_{max}$ are respectively the bounds of index number of the rays reaching the antenna mouth, which can present discontinuities. However the last rays close to the edge of the antenna mouth have to be weighted due to the fact the energy contain of such beam is different than the others depending on the ratio $(d_{oj}/d_{min})^{1/2} a_{oj}$ in 2D where $a_{oj}$ is initial amplitude associated to the $j^{th}$ sub-beam, $d_{min}$ is the sub-beam width and $d_{oj}$ is the distance of the received ray position in the $x=0$ plane to the antenna mouth aperture edge. In 3D this evaluation is quiet more complex due to the fact we have to consider the electromagnetic flux entering in the antenna aperture to determine the weighting factor. However if the number of rays (sub-beams) becomes higher better will be the accuracy without forgetting that this requires more computation time. At the end, a compromise should be found between the wanted precision and the computation time.

**b) test cases:**

Tests are presented to illustrate the different phenomenon arising during reflectometry experiments and their impacts on the measurements.

The first one corresponds to the destructive interference, which drives an extinction of the detected signal although the reflected signals exist as shown Fig. 2 for a possible configuration presented on the figure 1 (right).

![Figure 1: Ray tracing for 60 GHz probing frequency for linear density profile on O-mode with a standard radiation pattern (cardinal sine) where each is associated to the same energy flux by choosing the launch angle to fulfil this requirement. The rays with a light green colour correspond to those entering into the receiver. Only these light green rays contribute the amplitude and phase measurements. On the right the cut-off layer position is at $x_c = 1$ m and on the left two half linear density plasmas are attached to exhibit the destructive interference phenomenon as test case where one of the cut-off layer is moving.](image-url)
To illustrate this effect the density gradient length is changed to follow the measured intensity. On the figure 2, the 2D density profile evolution is given associated to the intensity evolution where the intensity is modulated as a function of the phase difference between the reflected wave coming from the top and bottom plasmas having different density gradient length. The intensity modulation is high in these due to the fact the phase difference is almost the same for the different sub-beams contributing to the intensity measurements despite the fact that the phase distribution over the receiver aperture is non-uniform (see Fig. 3). As the number of received rays stays constant, the only way to explain the intensity modulation is connected to the phase variations through interference processes.

![Figure 2: Intensity variations as a function of the phase difference (blue line) induced by the density gradient length changes (left), which are drawn (right) in the same probing conditions as Fig. 1. The arrows indicate the phase shift associated to the maximum and minimum of the intensity, and the line with red dots corresponds to the number of detected sub-beams.](image)

In fact only one part of rays contributes to the phase measurements, only those able to reach the antenna aperture are taken into account to evaluate the phase and amplitude measurements. In the figure 3 the phase evolution of each sub-beams are drawn showing that the spatial phase distribution has to be taken into account to interpret the phase and amplitude measurements.

![Figure 3: Total phase of each sub-beam as a function of their spatial distribution in the probing beam for one case of the figure 2. The introduction of an antenna aperture reduces the number of sub-beams contributing to the phase and amplitude measurements (dots encircled by red curves).](image)

The second test case deals with the average (measured) phase evolution as a function of the continuous changes of the density gradient length in plasma configuration shown on fig. 4. The spatial evolution of sub-beam phases shows that the central part doesn't change as expected. Only the rays at the edge see the plasma changes. However the measured phase has a continuous behaviour as function of the slope variations. The intensity behaviour is completely different due to the increased phase mixing, which drives a reduction of the coherency and consequently of the intensity. Again the number of the collected
sub-beams is constant so the intensity variations are only due to the mixing of sub-beam phases. That confirms the idea of the role of spatial phase distribution at the antenna mouth.

![Figure 4](image.png)

*Figure 4: (Bottom) Measured phase and intensity as a function of the plasma density slope, (Top) with an example of ray tracing for a given value of the slope and the associated sub-beam phases for the different slope values.*

The third test case looks for the dependencies of the antenna width in addition the slope variations (2nd test case). Intuitively, it is possible to say: when the width increases, the phase mixing increases inducing a reduction of the intensity as seen in the second test case. For this case, the main difference comes from the increasing number of collected sub-beams. Below a critical value corresponding to the a single slope case equal to $w_{rc} = 0.02$ m, the number of detected sub-beams having more or less the same sub-beams phase increases, that gives a constructive interference inducing a growth of the intensity (see Fig. 5 (right)). Above this critical width, the measured phase variations increase and a more complex situation arises with sub-beams presenting an additional phase jump, which becomes bigger for a larger antenna width that changes also the intensity behaviour (see Fig 5, right). Thus the intensity is more and more modulated with an increase of the lateral sub-beams, which is a consequence of the aperture width growth. Again the distribution of the sub-beam phase values is confirmed to be an important parameter to link the detected amplitude and the average phase. Another conclusion appears: a narrow antenna aperture permits to avoid or reduce the interference processes able to modify the amplitude measurements. But in this case, the amplitude measurements become more sensitive to the 2D effects (for example, cut-off tilting, blobs, islands, filaments, ....).

The fourth test case is linked to the role of the probing frequency at fix radiation pattern and same plasma parameters as in the previous test cases with slope equal to 1 are used. To illustrate this role a wide radiation pattern is used to enhance the dependency to the phase variations at the antenna mouth. Thus, the cut-off layer moves with the probing frequency that the reasons why the average phase values are changing with the probing frequency. At low frequency the phase excursion of the sub-beams stay below
2π, thus the interference phenomena stay small. The interferences begin to play a role when the phase difference of the sub-beams becomes of the order of 2π.

Figure 5: Measured phase and intensity as a function of the plasma density slope and the width of the antenna aperture with inputs common with the Fig. 4. The phase variations are computed for a slope variation between 0.95 and 1.05 for each width in the relevant range [0.016, 0.044] by step of 0.002 m.

At low frequency the phase excursions of the sub-beams stay below 2π, thus the interference phenomena stay small. The interferences begin to play a role when the phase difference of the sub-beams becomes of the order of 2π (see Fig.6). For a finite antenna aperture, knowing the probing expand more and more when it propagates into the plasma by increasing the probing frequency means that the reflected beam width increases during the frequency sweep due to the inward cut-off layer displacement. The effect is clearly seen on the fig 6 (right), for which the beam size becomes greater than the aperture width for a given frequency. The signature is simply given by the reduction of rays reaching the antenna aperture, and at the same time the phase mixing is reduced due to a decrease of the phase variation amplitude seen at the antenna mouth compared to an infinite aperture size. If the antenna aperture is reduced then in principle the critical frequency becomes higher. However in experiments the radiation pattern becomes wider at fixed frequency when the aperture size is reduced, thus a compromise should be found to optimize the reflectometer.

Figure 6: Measured intensity as a function of the probing frequency normalized to 60 GHz with a wide aperture antenna, that is to say able to collect all the reflected beams (left) and for a finite aperture width of 0.1m, which is no more able to capture all the rays due to a deeper inside cut-off layer permitting a larger expansion of the reflected beam.

The role of the phase at the antenna mouth has to be considered to know exactly what is the phase measured compare to the position of the reflection point (surface). If the phase difference along the antenna mouth becomes of the order of 2π then the measured intensity is affected and the link between phase and amplitude is difficult to establish. To interpret the measurement minimizing the error we have to answer to know what is the impact of the average done on the receiver. If this is a plane wave there is no effect, if it is spherical wave that introduces a phase shift correction to recover the true position of the cut-off layer.
2) **On the relevance of the optical modelling:**

The optical modelling consists to remove the wave propagation effects in plasma and just considering the cut-off layer as a corrugated mirror [1-3]. This modelling should provide information of the wave propagation coming back to the receiver. However, in my knowledge no body try to compare the behaviour of the wave reflected on a corrugated mirror with out and with plasma in front of it. On the Fig. 7, it is shown that the behaviour of the reflected wave is really different meanly due to the fact that the wave turns before reaching the cut-off layer.

![Figure 7: Ray tracing results for the optical modelling (Top) for a sine modulation of the metallic mirror surface, and (Bottom) the equivalent case where the vacuum is replaced by a linear density profile where the cut-layer is replaced by a metallic mirror.](image)

Thus it seems not clear at all that the optical modelling is an appropriate tool to describe the behaviour of a reflectometer signal. This is all the more true that the result depends on the density profile shape as shown on the Fig. 8 where the same density perturbation is used and defined by $\delta n/n_c = -0.05 \cos[6\pi(y - 1)] \exp[-300(x - 1)^2]$ with a cut-off position $x = 1 \text{ m}$.

Again the behaviour is different that those obtained using optical modelling, however, to interpret the shown results, we have to take care of the effective amplitude of the cut-off displacement. Looking at the phase and amplitude evolution as a function of the poloidal motion of the oscillation of the cut-off layer shows again (see Fig.9) that it is difficult to reconnect the results obtained by the optical modelling to those obtained in plasmas with different density profiles. Whatever it is hard to link the amplitude and phase behaviours to the input density perturbations when the amplitude of density fluctuations becomes greater than few %. At higher modulation amplitude, the results show that the detected intensity decreases with the increase of the amplitude modulation. However this decrease is much faster with a corrugated mirror than the equivalent plasma cases. Due to the multi-reflection the phase becomes more complex variations as a function of the poloidal displacement and the modulation is more detectable directly. As a partial conclusion the optical modelling seems to give more scattering than those done in plasmas. An explanation can be associated to the fact that the ray turns before reflection that reduces the angle and smooth the cut-off deformation effects. The phase mixing of the sub-beams is higher in vacuum than those in plasmas simply due to the fact that the wavelength increases near the cut-off layer, thus the intensity variations are more pronounced in the vacuum than in plasmas due to higher phase variations (see Fig. 9 bottom). To conclude, the optical model based on a corrugated mirror is not really appropriated to describe reflectometry measurements due to the fact this kind of modelling overestimates
the beam "scattering" on the cut-off layer. However, the models used are inappropriate to describe the Bragg backscattering effects or wave trapping or other non-linear effects, which can happen at high turbulence level [12,14,18].

**Figure 8:** Ray tracing results for the optical modelling (Top) for a sine modulation of the metallic mirror surface, and (Bottom) the equivalent case where the vacuum is replaced by a linear density profile where the cut-layer is replaced by a metallic mirror.

**Figure 9:** Comparison between two 2D ray tracing with plasma (right) and vacuum equivalent to the optical model (left) for a same poloidal modulation of the cut-off surface for measured phase variations (bottom), and the intensity and number of sub-beams reaching the detector.

### 3) Some studies on the link between phase and amplitude measurements:

**a) Small island:** An interesting case deals with the island detection in the case of small amplitude island. The phase variations at the antenna aperture at each case exhibit a small excursion. Thus the intensity follows more or less the number of sub-beams intercepting the antenna aperture, which is connected to the 2D geometry of the cut-off layer. This signature is given by the existing symmetry of the detected phase modulations on the antenna mouth. The phase evolution as a function of radial is different story
where the maximum value of the phase excursion is close to the cut-off layer. The shape and the amplitude of the phase responses are changing as the function of the island position compare to the cut-off layer position (see fig. 10 bottom). In the presented case, the islands are far away from the antenna, and if the island is closer to the antenna then the phase variations are more important on the antenna aperture this induces higher intensity modulations. Further studies should be done on this topic to evaluate what are the best conditions for the characterisation of magnetic islands take into account the geometrical effects.

Figure 10: Spatial (5 mm per step) and time evolution over one period of the response associated to an island (bottom left + zoom bottom right) modelled by a density perturbation with a small amplitude $\delta n/n_c = 0.005$ (top left) and the associated phase variations at the antenna aperture where colour corresponds to different radial positions (top right) with similar plasma conditions used previously.

b) Turbulence signature with narrow and wide radiation patterns for the detection.

Figure 11: Intensity and number of rays (left), and the phase (right) as a function of the turbulent wave front position in two cases narrow (continuous, light) and wide (dashed, dark) detector aperture with same plasma conditions and probing frequency of the other figures.
The intensity variations between wide and narrow detectors concerning the amplitude are those excepted due to the number of collected rays, for which the sub-beams have the same amount of associated power as it was defined previously. The intensities exhibit crudely the same behaviour but the effect of phase mixing can be clearly seen if one compares the number of rays and the intensity. For the narrow detector, both follow each other except in the cut-off layer. For the wide case is completely different due to the fact a lot of rays are mixed with a large range of phase variations. In the case of narrow, the phase behaviour is crudely the same with big variations close to the cut-off. However the phase variations are smoothed in the case of the wide detector. There is also a global phase shift between narrow and wide cases. As the phase seen corresponds to an average value and the equivalent wave front is curved, this shift is explained by these facts. However the phase shift introduces by the integration over the antenna mouth should be evaluated taken into account on the wave front curvature at the antenna mouth. That means it is better to have antennas far from the cut-layer to reduce the effect of phase mixing at the receiver.

Conclusions: The phase distribution on the collection surface of the receiver is a key parameter to provide relevant measurements. In this work we have ignored the contribution of the backscattered wave, which can have a random phase contribution but that has a very small impact of the phase mixing while the amplitude of the backscattered stays small. This situation is equivalent to few sub-beams with phase variations greater than $2\pi$. For high level of turbulence the phase-screen model is based on the same principle [19-20] but this model uses geometrical optics approximation and 2D full-wave simulation for a single value of poloidal wavenumber in [19] to explain the plasma curvature effects, however the evaluation of the impact in turbulent plasmas should be reconsidered using full-wave computations as also mentioned in [20]. Whatever a narrow probing beam seems to be the best choice to evaluate the amplitude and phase of the reflected wave, however this increases the probability to loose the reflectometer signal. Assuming an ideal synthetic antenna, it becomes possible to optimize the radiation pattern to reduce as possible the phase variation on the antenna mouth without losing the signal that should permit to measure the turbulence characteristics in the best conditions. In these conditions, the measures of the phase are smoothed in negligible manner and the amplitude is not affected by interferences associated to geometrical effects however it can be by scattering processes.

Bibliography: