Consideration Doppler reflectometry in Born approximation

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Doppler reflectometry is now extensively employed as an effective tool for plasma rotation measurements in toroidal devices [1]-[5]. The method is based on deriving of the rotation velocity from the Doppler frequency shift of backscattered radiation expected under an oblique incidence of microwave beam onto cutoff surface. It is assumed, that fair radial resolution of the diagnostics occurs due to microwave field enhancement in the vicinity of the cutoff. To verify this assumption 2D simulation of O-mode microwave backscattering was carried out in Born approximation. This simulation is valid only for small amplitude of fluctuations [6]. Nevertheless the considered simulation allows to reasonably determine a range of diagnostic parameters to proceed, if necessary, with a full-wave analysis. Developed 2D code is applicable for arbitrary radial profiles of background plasma density and for the scattering from any size and shape of plasma density fluctuations. The computation can be performed for any antenna tilt angle and for any phase and amplitude distribution of electric field across the antenna mouth.

An approach to numerical integration

In a frame of Born approximation it is possible to introduce spatial weighting function \( W(r) \) which links directly an output signal \( I(t) \) of IQ detector with plasma density fluctuations \( \delta n(r,t) \) [4].

\[
I(t) = \eta \int W(r) \delta n(r,t) \, d^2r \quad W(r) = \langle E_i(r,t)E_a(r,t) \rangle \quad (1)
\]

Where: \( \eta \) is a dimensional constant; \( E_i \) is electric field of incident beam and \( E_a \) is electric field of imaginary radiation launched into the plasma via receiving antenna; an averaging is performed over microwave period. The weighting function is computed with a full electric field of probing beam in a free space represented as a superposition of poloidal modes of convergent and divergent cylindrical waves.

\[
E_i(r,\varphi,t) = \sum_{m=-\infty}^{\infty} \left( \hat{E}(m)H^{(2)}_m(r) + \hat{A}(m)H^{(1)}_m(r) \right) \exp(i m \varphi - \imath \omega t) \quad (2)
\]

Here: \( H^{(1)}_m(r) , H^{(2)}_m(r) \) are Hankel functions; \( E(m) \) is \( m \) – spectrum of the incident beam obtained with use of prescribed distribution of electric field across the antenna mouth. The electric field in plasma is given by:

\[
E_i(r,\varphi,t) = \sum_m P(m)F_m(r) \exp(i m \varphi - \imath \omega t) , \quad (3)
\]

where: \( F_m(r) \) is a solution of one-dimensional wave equation. The complex spectra \( A(m) \) and \( P(m) \) are found by equalizing the vacuum electric field to the electric field on plasma...
boundary. The imaginary electric field $E_a$ of receiving antenna beam was obtained in a similar way.

**Spatial and wave number resolutions**

To quantitatively estimate spatial and wave number resolutions the two models of plasma density fluctuations have been employed given the weighting function is computed. The fluctuations are represented by a superposition of either variously directed plane waves of random phases with specified wave number $k_\perp$ or various poloidal modes. The fluctuations are localized in Gaussian shape layer at radius $r_0$. The first model represents isotropic plasma turbulence with correlation length of about reciprocal wave number $k_\perp$. The superposition of the poloidal modes is more relevant to plasma drift turbulence in tokamak plasma. The squared response $\langle I^2(r_0,k_\perp) \rangle$ or $\langle I^2(r_0,m) \rangle$ is calculated with use of expression (1) being averaged over different realizations with various sets of the random phases. The both radial $\Delta r$ and wave-number $\Delta k$ resolutions were taken at a half maximum of the 2D functions $\langle I^2(r_0,k_\perp) \rangle$ or $\langle I^2(r_0,m) \rangle$. With use of such an approach the influence of the cutoff curvature on the resolution has been studied. In Fig. 1 antenna size ($r_{\text{horn}}$) dependence of the k-resolution is shown for various radius of the cutoff surface $R_{\text{cutoff}}$. Qualitatively similar dependences were lately obtained with use of phase screen model [4]. The optimal antenna size corresponding to minimum of the wave number resolution is evidently seen for a moderate cutoff curvature. The increasing of the cutoff radius is followed by the improvement of the k-resolution. It is important that the spatial resolution has also a minimum at the same antenna size.

**Improvement of Doppler reflectometry resolution**

The observed degradation of the diagnostics resolution at lower cutoff curvature explicitly depends on width of the incident beam $m$-spectrum $E(m)$, as the eigenfunctions $F_m(r)$ are strongly dependent on poloidal number $m$. The width $\Delta m$ of the $m$-spectrum is given in paraxial approximation for Gaussian beam by

$$\left(\Delta m\right)^2 = r_{\text{horn}}^2 \left(\frac{r}{R} + 1\right)^2 + \frac{4r_{\text{horn}}^2}{r_{\text{horn}}^2} \left(\frac{r}{R} + 1\right),$$

(4)
where: \( r_a \) is radial coordinate of the antenna and \( R \) is radius of wave front at antenna mouth. The width of the \( m \)-spectrum has got a minimum at the antenna size corresponding to the minimum of the spatial and \( k \)-resolutions observed in Fig.1. On the other hand \( \Delta m \) has also got a minimum regarding radius \( R \). The minimum condition is satisfied at \( R_{\text{min}} = -r_a \). Therefore, it is natural to expect the improvement of resolutions at the concave wave front at antenna mouth. This has been checked by computation of the resolutions as a function of the wave front radius. The results are shown in Fig.2. The \( m \)-spectrum width \( \Delta m \) is also plotted here by dotted lines. The essential improvement of the resolutions occurred for the convergent beam with the predicted radius of the wave front. The effect of the improvement is more pronounced when the cutoff is placed in the Rayleigh zone that is for relatively large aperture of the antenna.

In line with theoretical prediction [6] the spatial resolution improvement might be for the convergent beam even at stratified distribution of the background density. In Fig.3 the resolutions are shown as a function the wave front radius for ITER relevant distribution of the background density at plasma periphery. The spatial resolution is somewhat improved at the wave front radius equal to a distance from the antenna to the cutoff. However the \( k \)-resolution turns out poorer than for the flat wave front. Therefore, a wave front radius reasonably follows from the trade off between the spatial resolution and \( k \)-resolution.

The fair resolution improvement can be achieved with use of two-antenna arrangement. The weighting functions computed for both monostatic and two-antenna arrangements are plotted in Fig.4 for comparison. One can see that with use of two antenna arrangement the probing and receiving antenna beams are not overlapped in the peripheral
regions leading to narrowing of the weighting function. The latter most probably results in fair improvement of the resolutions illustrated in Fig.5. Further increasing of a distance between antennas transfers the Doppler reflectometry to a collective scattering diagnostics, in which effect of refraction has not essential importance.

Summary and conclusion

The developed 2D code adequately describes the Doppler reflectometry in the conditions of tokamak experiment. Under a moderate cutoff curvature the resolutions may be improved significantly with use of the convergent microwave beams. The effect is more pronounced for relatively large aperture of an antenna. The code is appropriate for analyzing the two-antenna arrangement used to improve the diagnostics resolution. The problem still exists in proper consideration non-linear scattering. The influence of virtual distortion of beam fronts due to large amplitude MHD or/and ballooning modes is a disturbing factor to be analyzed in a first turn.

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Reference