Nonlinear effects in the Doppler reflectometry (analytical theory and numerical simulations)

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1. Introduction

One of widespread methods used at present for plasma poloidal velocity measurements is Doppler reflectometry [1–5]. This technique provides measuring fluctuations propagation poloidal velocity which is often shown to be dominated by plasma poloidal rotation velocity [3, 4]. This method is based on plasma probing with a microwave beam which is tilted with respect to the plasma density gradient. A back-scattered signal with frequency differing from the probing one is registered by a nearby standing or the same antenna. The information on the plasma poloidal rotation is obtained in this technique from the frequency shift of the backscattering spectrum which is supposed to originate from the Doppler effect due to the fluctuation rotation.

Spatial distribution of the scattering phenomena, which is usually assumed to occur in the cut-off vicinity, is the key issue for the diagnostic applications. In case of low turbulence level, when only single scattering is significant, this problem was investigated in the framework of linear theory numerically in [3] and analytically in [6–8]. The diagnostics locality in this case was shown to be better than for standard reflectometry. The approach used in [6–8] is based on the Born approximation. It assumed small enough amplitude of the fluctuations, which caused the scattering, to neglect a multiple scattering contribution to the received signal.

However this approximation can be incorrect in the plasma periphery where the turbulence amplitude is usually high, or in large plasma devices, where the probing ray trajectory is long. This is the case when the following criterion is satisfied [9, 10]

\[
\frac{\omega_i^2}{c^2} \ell_{cx} x_c \left( \frac{\delta n}{n_c} \right)^2 \ln \frac{x_c}{\ell_{cx}} > 1
\]

where \(n_c\) is the critical density corresponding to the probing frequency \(\omega_i\), \(x_c\) is the distance between the plasma and the cut-off, \(\delta n\) is the turbulence amplitude and \(\ell_{cx}\) is the turbulence radial correlation length. This situation of the high turbulent density perturbation level, when the multiple forward scattering of the probing wave is dominant, was investigated analytically in [11]. It was shown there that even in this regime the diagnostics is able to measure the plasma poloidal velocity with a spatial resolution close to that of standard fluctuation reflectometry. Here we present the comparison of the analytical results of [11] with the full-wave modelling of the Doppler reflectometry.

2. The model considered

The consideration is performed in the frameworks of the analytical approach [11]. Its limitations are the following. The plasma is assumed to be large: \(\lambda_0 \ll x_c\). The level of the turbulence is supposed to be modest

\[
\frac{\delta n}{n_c} < \frac{\ell_{cx}}{x_c}
\]

which provides the existence of the only one cut-off for the probing wave. The backscattering in radial and poloidal direction is neglected, which gives

\[
\ell_{cx} > \left( \frac{c^2 x_c}{\omega_i^2} \right)^{1/3}, \quad K \sin \theta > \ell_{cy}
\]
where $\ell_c$ is the turbulence poloidal correlation length, $c$ is the light velocity, and $K = \omega_i/c \cdot \sin \theta$ denotes the probing wavenumber poloidal component, where $\theta$ is the tilt angle. In addition, the analytical model is limited to the Gaussian antenna beams: $E \propto \exp \left[ -y^2/(2\rho^2) \right]$, where $\rho$ corresponds to the width of the antenna beam. All these assumptions allow us to take into account multiple small-angle scattering supposing arbitrary density and poloidal velocity profile, arbitrary turbulence spectra and statistically inhomogeneous turbulence.

For the sake of simplicity we consider the plasma slab with the linear density profile $n_e = n_c x/x_c$, perturbed by the statistically spatially homogeneous turbulence with Gaussian correlation function with characteristic length $\ell_c$:

$$\langle \delta n(x, y)\delta n(x', y') \rangle = \delta n^2 \exp \left\{ \frac{(x - x')^2 + (y - y')^2}{\ell_c^2} \right\}$$  \hspace{1cm} (2)

In the code the turbulence was represented by the set of harmonics with amplitudes, chosen to provide (2), and random phases. The poloidal plasma flow is assumed to be homogeneous

$$\frac{dv}{dx} = 0$$

The results of [11] for this simplified model can be represented as follows. The registered signal power takes the form

$$\langle |A_s|^2 \rangle = \frac{P_i}{4} \exp \left[ -\frac{2K^2}{1 + \sigma \frac{2\rho^2}{\ell_c^2}} \right] \left\{ 1 + \frac{c^2\bar{x}^2}{4x_c^2} \left( 1 + \sigma_1 \frac{2\rho^2}{\ell_c^2} \right) \right\}^{1/2}$$  \hspace{1cm} (3)

where $P_i$ is the incident power and $\sigma$ is the nonlinearity parameter [9]

$$\sigma = \sqrt{\frac{\omega_i^2}{c^2}} \ell_c x_c \left( \frac{\delta n}{n_c} \right)^2 \left[ 1 + \ln \frac{8x_c}{\pi \ell_c} - \frac{\gamma}{2} \right], \quad \sigma_1 = \frac{\omega_i^2}{c^2} \left( \frac{\delta n}{n_c} \right)^2 \ell_c x_c$$

and $\gamma \approx 0.577$ is the Euler constant.

One can easily see that due to the exponential suppression substantial signal can be registered only for the strong enough angular broadening of the probing beam

$$\Delta k_y \approx \frac{\sqrt{\sigma}}{\ell_c} \sim K$$

Other suppressing factors $(1 + 2\sigma \rho^2/\ell_c^2)^{-1/2}$, $[1 + (1 + 2\sigma_1 \rho^2/\ell_c^2) c^2 x_c^2/(4\omega_i^2 \rho^4)]^{-1/2}$ are associated with the beam angular and spatial broadening correspondingly.

In accordance with [11] the Doppler reflectometry spectrum can be represented as

$$S(\omega) = \frac{\sqrt{2\pi}}{\delta \omega} \exp \left[ -\frac{(\omega - \omega_i + \Delta \omega)^2}{2(\delta \omega)^2} \right]$$

where $\Delta \omega$ is the frequency shift

$$\Delta \omega = 2Kv \cdot \frac{2\sigma \rho^2/\ell_c^2}{1 + 2\sigma \rho^2/\ell_c^2}$$  \hspace{1cm} (4)

which takes the form $\Delta \omega = 2Kv$ for $\sigma > \ell_c^2/(2\rho^2)$ and $\delta \omega$ is the spectral broadening:

$$(\delta \omega)^2 = \sigma \left( \frac{\langle \Omega^2 \rangle + \bar{v}^2}{\ell_c^2} \right) \cdot \frac{2}{1 + 2\sigma \rho^2/\ell_c^2}$$
3. Full-wave simulation parameters and results

The probing wavelength was taken as $\lambda_i = 0.75$ cm, which corresponds to the probing frequency $f_i = 40$ MHz and critical density $n_c \simeq 2 \cdot 10^{13}$ cm$^{-3}$. The probing beam poloidal structure is represented in the figure 1.

The electric field in the plasma was calculated in the 2D rectangular area with dimensions $L_x = 150\lambda_i$, $L_y = 125\lambda_i$ in the presence of the homogeneous turbulence with correlation length $\ell_c = 4.5$ cm. The calculation results for different turbulence amplitudes are shown in the figure 2. It can be noted that the probing beam is completely distorted by the turbulence in case of $\delta n/n_c = 10\%$ (figure 2(b)).

The registered signal power is plotted in the figure 3(a) via turbulence amplitude. One can see the correspondence between the analytical expression (3) and the numerical calculation results for moderate turbulence amplitudes. Fast growth of the signal at the small turbulence amplitudes is associated with the decrease of exponential factor influence in (3). Following decrease of the signal corresponds to the influence of the denominators in (3), discussed above. The divergence of analytical formulae and numerical results in this region can be associated with the violation of the criterion (1).

The stationary structure, modelling the turbulence, moved in the poloidal direction with a velocity $v = 28.5$ km/s, imitating the plasma poloidal rotation. This results in the Doppler shift of the registered signal, which is plotted in the figure 3(b). It can be seen that the dependence of the spectrum shift on the turbulence amplitude, predicted by the analytical formula (4), is reproduced in the simulation results.
4. Conclusion

Doppler reflectometry is considered in the nonlinear regime of dominating multiple forward scattering. Analytical expressions and full-wave simulation results for the power and spectrum shift of the registered signal are compared and shown to be in agreement in the region of analytical theory applicability.

The possibility to measure the fluctuation poloidal velocity with Doppler reflectometry technique even in regime of strong small-angle multi-scattering typical for large reactor-scale experiments is confirmed.

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References