2D full wave simulation of poloidal and radial sensitivity of reflectometer and estimation of density fluctuations absolute level.

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I. Introduction

A lot of attention is devoted to the investigations of plasma transport now. Recent theories claim the turbulent transport as the reason of an anomalous transport in tokamak plasma. The main instabilities that should be the responsible for the transport in plasma core are Ion Temperature Gradient (ITG) instability\(^1\) and Dissipative Trapped Electron Mode (DTEM)\(^2\). The character poloidal wave numbers for there instabilities are \(k_\perp \times \rho_i \sim 1\) that give the character poloidal side of instabilities about 1 cm.

It was shown experimentally that reflectometry is a very sensitive to small-scale perturbations of plasma density\(^3\). Application of the correlation reflectometry technique allows to measure of character size of turbulence and its velocity. Yet non-locality of reflectometry measurements significantly complicates the interpretation of results.

This paper devoted to attempts to reveal the possibility of using reflectometry as a diagnostic for the measurements of the local parameters of density fluctuations. It became possible with the development of analytical methods as well as 2D full wave codes. The first part of the paper dealing with the 2D simulation of the O-mode reflectometry measurements in real T-10 experiments geometry and the rest of the paper related to the comparison of results of simulations with full wave codes and analytical approaches. The obtained results are discussed in conclusions.

II. 2D full wave simulation of reflectometry measurements.

One of the most important topics in reflectometry simulation is the set of perturbed density field. It was used the model proposed for the simulation of reflectometry at plasma periphery\(^4\). Due to this approach, the stochastic density field is calculated as:

\[
\bar{n}_e(r, \phi, t) = n_e(r, \phi) + \sum \delta n_e(r, \phi, t)
\]  

(II.1)

where \(r\) is the minor radius, \(\phi\) – poloidal angle and \(t\) – time. The first term corresponds to the time-averaged density profile, the second one – to density perturbations. The density perturbations are given by superposition of stochastically independent fluctuations:

\[
\delta n_e(r, \phi, t) = n_0 \times T(t) \times R(r) \times \Phi(\phi, t)
\]  

(II.2)

\[
T(t) = \left[ \exp\left( -\frac{(t-t_0)}{t_{\text{decay}}} \right) - \exp\left( -\frac{(t-t_0)}{t_{\text{rise}}} \right) \right] \times H(t-t_0)
\]  

(II.2.1)

\[
R(r) = \exp\left( -\frac{(r-r_0)}{\Delta_r} \right)^2
\]  

(II.2.2)

\[
\Phi(\phi, t) = \exp\left( -\frac{\left((\phi - (\phi_0 + \Omega_\perp \cdot (t-t_0))) \cdot r_0 \right)}{\Delta_\perp} \right)^2 \times \left(a + b \cdot \cos(k_\perp \cdot (\phi - (\phi_0 + \Omega_\perp \cdot (t-t_0))) \cdot \Delta_\perp) \right)
\]  

(II.2.3)

The first term is the amplitude of single perturbation, second one is time evolution of single perturbation, third and fourth – radial and poloidal shapes of perturbations. The perturbation is appears in time moment \(t_0\) with center situated at position \(r_0, \phi_0\), and has character time of rise \(t_{\text{rise}}\) and decay \(t_{\text{decay}}\). Character radial and poloidal size are characterized by Gaussian width \(\Delta_r\) and \(\Delta_\perp\). Plasma supposed to rotate in poloidal direction with angular velocity \(\Omega_\perp\). Coefficients \(a\) and \(b\) give relative amplitudes of Broad Band (BB) and Quasi Coherent (QC) oscillations amplitude in spectra respectively. The poloidal wave number for QC fluctuations is \(k_\perp\). Values of perturbation parameters were chosen close to the experimental ones and varied along the minor radius.
This stochastic plasma density field was re-calculated in plasma permittivity field that was used in full wave simulation. High effective 2D full wave electromagnetic code Tamic Rτ Analyzer was used to calculate the propagation and scattering of electromagnetic waves from plasma. The T-10 plasma and antenna array geometry were used in simulations. Electromagnetic wave launched from the central horn and reflected signals from all three poloidally separated horns were analyzed.

The most of simulations were made using the following parameters: reflectometry frequency $F = 37$ GHz that corresponds to reflection at normalized minor radius $\rho = 0.65$, character length of plasma permittivity profile $L_{\epsilon} = (d(\ln(n_e))/dr)^{-1} = 16$ cm, fraction of QC oscillation was 15%, the turbulence parameters are shown in Table 1.

Locality of reflectometry was characterized by the cross-correlation coefficient between received electric field vector $\vec{E}(t)$ and perturbed local density:

$$\gamma(r,\varphi) = \frac{\langle \vec{E}(t) \times \vec{n}_e(r, \varphi, t) \rangle}{\langle |\vec{E}(t)|^2 \rangle \times \langle |\vec{n}_e(r, \varphi, t)|^2 \rangle}$$  (II.4)

The contour plots of this coefficient, calculated for all three poloidally separated antennas, are shown in Figure 1. The confidence level is 0.06% and determined by a limited length of data sequences. One should note that coherency maxima are laid at the reflection radius, shown by the horizontal black lines. The poloidal positions of coherency maxima are in good agreement with the positions of reflection spots for all three horns (black vertical lines). This means that reflectometry measurements have a good locality in both radial and poloidal directions and could allow providing the fluctuations size and velocity measurements.

The experimental radial correlation function is shown on Figure 2 a. The measurements were made in T-10 tokamak, discharge parameters were $I_p = 155$ kA, $B_T = 2.0$ T, $<n_e> = 2.5 \times 10^{19}$ m$^{-3}$, reflection occurred at $\rho=0.65$. The radial correlation reflectometry technique was obtained by simulation of wave propagation at the different probing frequencies on the same turbulent density field. Turbulence properties were as shown in Table 1, except only BB fluctuations were used. The results are shown in Figure 2 b) as red points and approximating curve. Radial cross-correlation function of density perturbation at the reflection radius is shown on the same plot as green curve. The parameters of turbulence were used to compare the experimental

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Expression</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{BB}^\rho$</td>
<td>0.7 cm</td>
<td>$\Delta^\rho \approx \frac{\Delta \rho}{3}$</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta_{BB}^\perp$</td>
<td>0.7 cm</td>
<td>$\Delta^\perp \approx \frac{2}{\Delta (Y_{BB}^\perp (k_i))}$</td>
<td>0.61 cm</td>
</tr>
<tr>
<td>$\Delta_{QC}^\perp$</td>
<td>0.7 cm</td>
<td>$\Delta^\perp \approx \frac{L_{\epsilon}^\perp}{3}$</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta_{QC}^\rho$</td>
<td>4.0 cm</td>
<td>$\Delta^\rho \approx \frac{2}{\Delta (Y_{BB}^\rho (k_i))}$</td>
<td>3.8 cm</td>
</tr>
<tr>
<td>$\lambda_{BB}^\perp$</td>
<td>2.1 cm</td>
<td>$\lambda_{BB}^\perp \approx 2\pi/k_a$</td>
<td>2.1 cm</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$2.46 \times 10^5$ cm/s</td>
<td>$\nu = \Delta x / \Delta \tau$</td>
<td>-</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-</td>
<td>(III.7)</td>
<td>0.35</td>
</tr>
<tr>
<td>$\sigma_n/n$</td>
<td>0.75 %</td>
<td>(III.10)</td>
<td>0.81 %</td>
</tr>
</tbody>
</table>

Table 1. Character parameters of turbulence at the reflection radius in 2D full wave simulations, expressions used in estimations of these parameters from experimental signal spectra and values parameters estimated from simulated specter.

Figure 1. Contour plots of correlation coefficient between scattered electric field and local density.
Radial cross-correlation function was calculated to using the same turbulence parameters as in 2D simulation (Figure 2 c).

It is clearly seen that experimental, simulated and theoretical radial cross-correlation functions of reflectometry signals are in reasonable coincidence. Each function contains some core that presents the cross-correlation function of a density perturbations and long tail that arise from small angle scattering. Both simulation and theory justify the fact that reflectometry has a good radial locality and could give estimation of perturbation radial size.

Other important topic is the turbulence spectra measurements. Size of reflection spot could significantly exceed the fluctuations size. It could lead to decrease of reflectometry sensitivity to perturbations with small spatial size. Typical experimental spectra in T-10 discharge is shown in Figure 3 a. The parameters was \( I_p = 200\, \text{kA}, B_T = 2.5\, \text{T}, \langle n_e \rangle = 2 \times 10^{19}\, \text{m}^{-3} \), reflection occurred at \( \rho = 0.65 \). One could see that spectra has a complex structure and contains Low Frequency (LF) and QC oscillations maxima that are superimposed on BB background.

First simulation run was made using BB turbulence only (Fig. 3 b). The simulated spectra of the reflected signal (red) and spectra of density fluctuations at the reflection radius (green) are in good agreement. This leads to conclusion that at least for BB oscillations reflectometry sensitivity is not important for interpretation of measurement results.

Second run used both BB and QC perturbation in random density field (Fig. 3 c). It is clearly seen that contrast of QC fluctuation peaks is significantly higher in spectra of local density rather than in spectra of reflected signal. Therefore, reflectometry has a limited sensitivity to QC oscillations and this fact should be taken into account in data processing.

Simulation of the poloidal correlation properties of turbulence shows that not only experimental and simulated turbulence spectra but also poloidal cross-phase and poloidal coherency are in good quantitative and qualitative agreement. It should be especially note that the slope poloidal velocity calculated from slopes of experimental and simulated cross-phase are coincide with the poloidal velocity of turbulence at the reflection radius. This leads to conclusion that estimation of poloidal velocity using the poloidal correlation could give the correct results.

### III. Estimation of the relative amplitude of density perturbations

The results of 2D simulation of reflectometry showed that reflectometry has a rather good radial locality and sensitivity to perturbations with character sizes that are predicted for ITG and DTEM. It is possible to reconstruct the density fluctuations level at reflection point.

The simplest case will be considered. Let us suppose that O-mode reflectometry signal reflected from results with analytic estimations. Radial cross-correlation function was calculated to using the same turbulence parameters as in 2D simulation (Figure 2 c).

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The simplest case will be considered. Let us suppose that O-mode reflectometry signal reflected from
the plasma with linear density and plasma permittivity profile with character length \( L_e \). It was shown that in 1D geometric optics approach spectral power of radial correlation function of density perturbation \( \Gamma_n \) and signal phase \( \Gamma_\phi \) are related by the following equation:

\[
\Gamma_\phi(k_r) = 2\pi \frac{k_0^2 L_e}{k_r} \left[ C^2(w) + S^2(w) \right] \Gamma_n(k_r) \quad (III.1)
\]

where \( k_0 = 2\pi/\lambda_0 \) is the wave number of incident electromagnetic wave, \( C(w) \) and \( S(w) \) — Fresnel integrals and \( w = \frac{1}{2} k_r L_e / \pi \). In a case of Gaussian, radial shape of perturbation the power spectrum of radial correlation function has a Gaussian form with character width \( \Delta^\text{cor}_r = \Delta_r \times \sqrt{2} \). Integration of both part of equation (III.1) using Gaussian spectra form leads to the following relation between the amplitude of phase fluctuation \( \sigma_\phi \) and relative amplitude of density perturbations \( \sigma_n/n \):

\[
\sigma_\phi = \left(\frac{\phi}{2}\right)^{1/2} = 4\pi K_1 \left\{ \frac{\Delta^\text{cor}_r}{L_e} \right\} \frac{\sigma_n}{\lambda_0} \frac{\sigma_n}{n_{er}} \quad (III.2)
\]

Coefficient \( K_1 \) is of order of unity and depends only on the ration of character size of turbulence and permittivity profile length. Numerical integration was used in a wide range of parameter to obtain the following approximation:

\[
K_1 \left\{ \frac{\Delta^\text{cor}_r}{L_e} \right\} = \sqrt{\int_0^\infty \left\{ C^2(w) + S^2(w) \right\} \exp \left( -\pi^2 w^4 / 16 \left( \frac{\Delta^\text{cor}_r}{L_e} \right)^2 \right) dw} \approx \sqrt{0.25 \ln \frac{L_e}{\Delta^\text{cor}_r} + 0.58} \quad (III.3)
\]

This approximation is similar to derived analytically, but numerical coefficient are different.

1D full wave simulation was used to verify derived expressions. Density perturbations had uniform distribution with \( \sigma_n/n = 0.5 \% \). The results are presented in Figure 4 as a comparison of the phase perturbation of simulated signal with the theory predictions. One could see that in a case of weak turbulence with uniform distribution 1D geometric optics approach could give a good estimation of the perturbation amplitude at reflection radius.

The estimation of reflectometry sensitivity to perturbations with high poloidal wave number \( k_\perp \) could be made using phase screen model. The perturbations of density at the reflection surface are considered in this case as a perturbation of electric field on a phase screen, situated at effective optical distance from launched and receiving horns \( d \). Let us suppose that plasma has a poloidal cross-section with center in origin of coordinates, incident beam has a Gaussian distribution of electric field with effective size \( w \) and front curvature of beam when it incident on plasma is \( r_0 \). If plasma permittivity profile is linear and horns situated in horizontal plane at \( z_0 \), the distance from horns to phase screen is equal \( d = (z_0 - r_k) + L_0 \), where \( r_k \) is the radius of reflection surface. Electric field in this case has a following distribution on the phase screen:

\[
E_\perp(\xi) = \frac{1}{\sqrt{2\pi w}} \exp \left( -\frac{\xi^2}{2w^2} \right) \times \exp \left( ik_0 \frac{\xi^2}{2w^2} \rho \right) \times \exp \left( \frac{\sqrt{2} \sigma_\phi \cos(k_\perp \xi + \theta)}{\rho_{\text{max}}} \right) \times \left( 1 - \frac{\xi^2}{\rho_{\text{max}}^2} \right) \quad (III.4.1)
\]

\[
\rho_{\text{eff}} = \left( \frac{\rho_0}{(\rho_0 + L_0)} + \frac{1}{\rho_0} \right)^{-1} \quad (III.4.1)
\]

\[
\rho_{\text{max}} = \left( \frac{1}{\rho_0} + \frac{1}{\rho_0 + L_0} \right)^{-1} \quad (III.4.2)
\]

The first term is the distribution of the electric field in incident beam, second one arises from the curvature of the plasma surface and beam front, third one is the modulation of electric
field by the harmonic density perturbations and last one is the attenuation of the field due to oblique incidence on the reflection surface. The electric field in receiving horn can be calculated using Fresnel-Huygens formula. In weak turbulence regime with $\sigma_0 \ll 1$ and $k_\perp \ll k_0$ after averaging over $-\pi < \theta < \pi$, the estimated relative perturbation of electric field in receiving horn is given by the following integral:

$$\frac{\sigma_2D(k_\perp)}{\sigma_{2D}(0)} = F_2(k_\perp) = \int_{\rho_{\max}}^{\rho_{\min}} \frac{1}{2\pi w} \exp\left(-\frac{\rho^2}{w^2}\right) \times \exp\left(\frac{i k_0 \rho^2}{p_\text{eff}}\right) \times \cos(k_\perp \xi) \times \exp\left(\frac{i k_\perp \rho^2}{2d}\right) \times \left(1-2\frac{\rho^2}{p_{\max}^2}\right) d\rho$$  (III.5)

We take into account also that in small $k_\perp$ limit perturbation of electric field tends to the value of phase perturbation in 1D approach. If the radius of the reflection surface significantly exceeds the beam width and $L_e \rightarrow 0$, equation resulted in analytical estimation.

2D full wave simulations were made to justify the phase screen model. T-10 antennas geometry was used, harmonic oscillations with different $k_\perp$ and relative amplitude $\sigma_n/n = 0.5\%$ were placed at different reflection positions and resulted electromagnetic filed perturbations were analyzed (Fig. 5). One could see that phase screen model gives good estimation of reflectometer sensitivity.

If turbulence spectra contains several components with different character size it should be take into account. Let us suppose that lifetime of perturbation is infinite. In this case, the frequency spectra could be re-calculated in $k_\perp$ spectra (Fig. 6). Using the perturbation shape (II.2.3) it is possible to estimate the character spatial size of turbulence (Tab. 1). The experimental specter (black) in this case can be presented as a sum of statistically independent QC (red) and BB (green) components:

$$Y_{2D}(k_\perp) = \sqrt{(1-\mu)Y_{BB}^{2D}(k_\perp)^2 + \mu Y_{QC}^{2D}(k_\perp)^2}$$  (III.6)

where $\mu$ is the effective fraction of QC in signal. The value of $\mu$ could be estimated as:

$$\mu = \frac{\kappa^2}{1+\kappa^2}$$  (III.7.1)

$$\kappa = 2 \frac{\Delta_{BB}^2}{\Delta_{QC}^2} \frac{Y_{QC}^{2D}(k_{\perp_{max}})}{F_{12}(k_{\perp_{max}})^2 Y_{BB}^{2D}(0)}$$  (III.7.2)

where $Y_{2D}^{QC}(k_{\perp_{max}})$ and $Y_{2D}^{BB}(0)$ are the amplitudes of QC and BB components in spectra. If $Y_{2D}$ is the experimental specter of turbulence and $Y_{1D}$ is specter in a limit of infinite probing frequency, they are related as:

$$Y_{2D}(k_\perp) = Y_{1D}(k_\perp) \cdot F_{12}(k_\perp)$$  (III.8)

The effective correction for limited reflectometer sensitivity calculated as follows:

$$K_2 = \sqrt{\frac{\int Y_{1D}(k_\perp)^2 dk_\perp}{\int (Y_{1D}(k_\perp) \cdot F_{12}(k_\perp))^2 dk_\perp}}$$  (III.9)

The level of density perturbations is estimated according to (III.2) taking into account the different radial scale length of BB and QC oscillations. In a case of statistically independent fluctuations, it leads to following equations.
Parameter $\Delta_{\text{eff}}^\rho$ has a sense of effective radial length of perturbations.

This approach was used to estimate fluctuations parameters from the reflectometer signals, obtained in 2D simulations. The density fluctuations parameters in this case are known and it is possible to compare them with the estimated ones (Tab. 1). One could see reasonable agreement between simulation parameters and estimations.

The method was used to estimate turbulence level in T-10 discharges with Ohmic and ECR heating. The results are shown in Figure 7. The level of density perturbations is increased in ECRH discharges in accordance with confinement degradation. It should be noted that $K_2$ coefficient slightly varied along the minor radius and its value was close to unity. Therefore, reflectometer sensitivity to high $k_\perp$ does not effect on the measurements.

**IV. Conclusions**

The using of reflectometry as a method of turbulence measurements was considered by means of 2D simulation and various analytical approaches. It was found that in a case of weak turbulence both simulations and theory shows good locality of this method. Both simulation and phase screen model estimations shows that reflectometry has a good sensitivity to perturbations with $k_\perp$ up to $k_0/3$. Therefore, reflectometry can measure spatial size, velocity and relative amplitude of fluctuations at the reflection radius.

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