Adaptive window calculation for automatic spectrogram analysis of broadband reflectometry data

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Abstract

Electron density profiles of fusion plasmas may exhibit large variations due to different operating regimes as well as fast changes due to transient plasma phenomena (e.g., ELMs), which modify the frequency content of broadband reflectometry signals. For this reason, the evaluation of the density profile based on the spectrogram technique usually requires: (i) the use of conservative sizes of the window of analysis, in order to cope with the large/fast data variations, or (ii) direct user intervention to match the window size to the data content. In the first case, accuracy is reduced, while in the second, the need for user intervention prevents the fully automatic evaluation of the density profile. Here we present the application of a new technique that uses a time-frequency energy concentration measure in order to estimate the optimum window length. In this way, the spectrogram analysis may be automatically adapted to the frequency content of the data in each window, increasing the time-frequency resolution in cases where the frequency exhibits fast/large variations, as for example just inside the pedestal region of H-mode plasma profiles. The application to test signals and simulated profile data shows the importance of the new technique in improving the accuracy of automatically evaluated density profiles. The advantages of the technique versus its computing requirements are also discussed.

1 Introduction

Broadband reflectometry as a diagnostic of the electron density profile of fusion plasmas as reached a level where profile availability to the physics community is of crucial importance for the full establishment of the diagnostic technique. In this context, automatic data evaluation procedures play a major role, since they allow the density profiles to be routinely available – depending on the characteristics of the diagnostic (e.g., the amount of data to be processed, available computing power, etc.) the profiles might even be available immediately after each shot. However, the automatic evaluation of the density profile from broadband data faces a major problem: due to profile modifications the frequency content of the reflected signals (directly related to the group delay of the probing waves) varies significantly which requires adjustments of the data processing parameters, that vary depending on the specific technique used in the evaluation.

In the case of the spectrogram analysis used to evaluate the profile data on ASDEX Upgrade, the parameter to be adjusted is the length of the window of analysis. By varying the size of the spectrogram window, it is possible to adjust the time-frequency resolution to a specific feature of the signal being analysed. However, in cases where the frequency content exhibits fast/large variations along the sweep a fixed-length analysis window is often not adequate to provide the required time-frequency resolution. In these cases, the evaluation of the density profile based on the spectrogram technique requires: (i) the use of large windows, in order to cope with the fast data variations, or (ii) direct user intervention to match the size of the window to the data. In the first case, accuracy is reduced, while in the second, user intervention disables the fully automatic evaluation of the data.

Here, we report on the application of a new technique that allows the length of the window of analysis to be automatically adjusted to the frequency content of the signals being analysed. This is achieved through the use of an energy concentration measure in the time-frequency plane. This measure is applied at each time sample to a given set of possible lengths of the window of analysis, in order to determine the optimum window length. The technique, first proposed in [1], is based on the assumption that the optimum window length (the one that results in good time-frequency localization and high resolution) is the one which provides the higher time-frequency concentration.
2 The adaptive spectrogram

2.1 Basic principles

Adaptive time-frequency representations require a means of determining an appropriate window length (in the case of the spectrogram analysis) or kernel function (in the case of other distributions) without extensive a-priori knowledge of the characteristics of the signal being analysed. The adaptive spectrogram technique presented here optimizes the length of the window of analysis for each time slice by maximizing a short-time measure of time-frequency concentration defined as

\[
C(t, p) = \frac{\int_{-\infty}^{\infty} |D_p(\tau, \Omega)w(\tau - t)|^2 d\tau d\Omega}{(\int_{-\infty}^{\infty} |D_p(\tau, \Omega)w(\tau - t)|^2 d\tau d\Omega)^{\frac{1}{2}}}. \tag{1}
\]

In the above, \(D_p(\tau, \Omega)\) is the spectrogram distribution and \(w(\tau)\) is a one-dimensional concentration window centered at \(\tau = 0\). The optimal time-varying window length is thus defined as \(p^*(t) = \arg \max_p C(t, p)\). Notice that the ratio of the \(L_4\) and the \(L_2\) norms of the spectrogram used in the concentration measure favors peaky distributions that place as much signal energy into as small a region of time-frequency as possible.

The implemented algorithm works as follows. The short-time concentration is computed for discrete values \(p_i, i = 1, \ldots, P\) of the window length. The resulting concentration measures \(C(t, p_i)\) represent samples of \(C(t, p)\). An estimate of the optimal window length \(p^*(t)\) is then obtained by taking the maximum value of \(C(t, p_i)\). Once the optimal window length \(p^*(t_0)\) is computed at time \(t_0\), the time slice \(D_{p^*(t_0)}(t_0, w)\) of the adaptive spectrogram can be computed with window length \(p^*(t_0)\).

The algorithm can be implemented more efficiently by rewriting \(C(t, p)\) in terms of the quantities

\[
c_2(\tau, p) = \int_{-\infty}^{\infty} |D_p(\tau, \Omega)|^2 d\Omega \quad \text{and} \quad c_4(\tau, p) = \int_{-\infty}^{\infty} |D_p(\tau, \Omega)|^4 d\Omega
\]

and taking advantage of the localization effect of \(w(\tau)\). More details of the implementation can be found in [1].

2.2 Application example

In order to evaluate the performance of the new technique we applied it to the test signal shown in figure 1(a). The signal has 256 samples and is composed of three diracs (located at samples 10, 20, and 250), a sum of two fixed-frequency signals (limited to between samples 45 and 95), a fixed-frequency signal modulated by a gaussian pulse (centered at sample 129), and a linear chirp with frequencies going from \(f = 0.05\) at \(n = 157\) to \(f = 0.3\) at \(n = 220\).

Figures 1(b)–(d) show the spectrogram distribution computed with window lengths of 5, 15, and 41 samples respectively. As can be observed, the small window is able to clearly localize the dirac pulses but fails to reproduce the other frequency components. Using a larger window enables us to recognize the existence of the other frequency components although the time-frequency resolution is rather poor. In addition, the dirac pulses are also poorly resolved. Finally, a large window is able to resolve all the components except the dirac pulses, due to the resulting poor time resolution.

Figure 1(e) shows the adaptive spectrogram distribution of the test signal computed using the technique described in 2.1. As can be observed, the adaptive spectrogram is able to resolve all frequency components with good time-frequency resolution throughout the signal. The corresponding evolution of the optimum window length is shown in figure 1(f). As expected, a small window was used in the region where the diracs are present, providing good time resolution. As soon as the sum of two frequencies is detected a large window is used in order to provide the necessary frequency resolution. A smaller window is used between samples 157 and 220 in order to resolve the frequency evolution of the linear chirp component.

3 Application to simulated profile data

For the ASDEX Upgrade broadband reflectometer, the application of the adaptive spectrogram to the automatic evaluation of density profiles has a major impact in suppressing the need to adapt the window length to the characteristics of the reflected signals. This is especially true during sudden changes in the profile gradient such as the ones occurring just inside the pedestal region in H-mode plasmas, during ELMs or due to changes in plasma density. Currently, the
window length is (manually) fixed at 256 samples for the K, Ka, and Q channels and 128 for the V and W channels, where the pedestal region is expected to be in medium and high density H-mode plasmas respectively. Although adequate for most cases, this setup does not provide good time-frequency resolution in the low/medium density part of the discharges (such as before the L-H transition) where the profile flattening might occur in the Q and even in the Ka channels. In these cases it would be best to use smaller window lengths in order to provide enough time-frequency resolution to resolve the profile flattening.

To evaluate the effect of using a large window of analysis in the region where the gradient is changing rapidly we used V-band data obtained from a simulated profile of the form

\[ n_e(r) = (1 - (r/a)^n)^m \tag{2} \]

with \( n = 2 \) and \( m = 4 \). The raw signal is shown in figure 2. As expected, the beat frequency exhibits a sudden increase due to the profile flattening above \( f = 70 \text{ GHz} \). The spectrograms shown in figures 2(b)–(d) were computed with 64, 128, and 256 samples respectively. As can be observed, a large analysis window leads to an unacceptable decrease in the time and frequency resolution in the region where the beat frequency is changing fast. As a result, the spectrogram distribution is unable to reproduce the correct frequency behaviour. The corresponding adaptive spectrogram is shown in figure 2(e) while the evolution of the window length across the signal is depicted in figure 2(f). In the region where the beat frequency is changing slowly the adaptive technique estimated a large window providing essentially frequency resolution. As soon as the beat frequency starts to increase the window becomes smaller and smaller to provide the necessary time resolution. Notice also the clear improvement in frequency resolution in the range below 70 GHz due to the use of a larger window.

From these results one can conclude that the adaptive spectrogram is an efficient technique to automatically adapt the length of the window of analysis to accommodate for possible beat frequency changes in broadband reflectometry data.

4 Discussion

The adaptive spectrogram technique allow us to automatically adapt the window length parameter to the characteristics of the reflectometry signals. By matching the window of analysis to the local time-frequency properties of the signals, both time and frequency resolutions can be simultaneously and independently improved. As such, large/fast beat-frequency variations induced by profile changes may be resolved, directly improving the accuracy of the inverted profile. However, as expected, extra performance doesn’t come without a price: the adaptive spectrogram requires about six times the computation time of a fixed-window spectrogram. Due to this additional computational effort, the use of this technique in the routine evaluation of density profiles on ASDEX Upgrade will be limited to the frequency band “containing” the profile flattening (the so-called last band). This alone represents a major improvement in the accuracy of level-1 profiles (profiles computed with a minimum of data processing effort) since it provides a means for adapting the data analysis parameters to changes in the electron density throughout the discharge.

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References

Figure 1: Application of the adaptive spectrogram technique to a test signal: (a) test signal, (b) 5 samples window, (c) 15 samples window, (d) 41 samples window, (e) adaptive spectrogram, and (f) window length evolution.

Figure 2: Application to simulated V-band data in the profile flattening region: (a) simulated signal, (b) 64 samples window, (c) 128 samples window, (d) 256 samples window, (e) adaptive spectrogram, and (f) window length evolution.